

# Comparison of Large-Signal-Network-Analyzer Calibrations

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**Abstract**— We develop a procedure and metrics for comparing large-signal-network-analyzer calibrations. The metrics we develop provide a bound on differences between measurements obtained from large-signal-network-analyzer calibrations, as well as more specific information on how the power, phase, and scattering-parameter portions of the calibrations differ.

**Index Terms**—calibration, comparison, large-signal network analysis, scattering parameters.

## I. INTRODUCTION

WE develop a procedure and metrics for comparing large-signal-network-analyzer (LSNA) [1-3] calibrations. The approach is closely related to the “calibration comparison” method of [4], which is commonly used to compare differences between conventional vector-network-analyzer calibrations and test-set drift [4], to determine characteristic impedance [5] and to measure the permittivity of thin films [6].

The calibration comparison method is based on a pair of “error boxes” relating two calibrations. These error boxes are most commonly determined by performing a “first-tier” calibration of the vector network analyzer and then using it to correct measurements of the calibration standards used to perform the “second-tier” calibration. The second-tier calibration calculates two scattering-parameter error boxes that map measurements corrected by the first-tier calibration into measurements corrected by the second-tier calibration, and allow the calibrations to be easily compared.

If the first-tier and second-tier calibrations are exact, the transmission matrices describing these error boxes are equal to the identity matrix. The extent to which the transmission matrices of the two error boxes relating the two calibrations differ from the identity matrix is used to develop a metric bounding the differences of the scattering parameters of passive devices measured by the two calibrations [4].

While vector network analyzers are designed to measure scattering parameters, LSNA are designed to measure the magnitude and phase of the amplitude coefficients of the forward and backward waves at each port of a device under

test [1-3;7]. Here, we extend the calibration comparison method to LSNA calibrations [8;9]. We develop a simple metric that quantifies the differences of wave amplitude coefficients corrected with different LSNA calibrations, as well as differences in the power, phase, and scattering-parameter portions of the calibrations. While this is not a substitute for a complete error analysis, it does offer a straightforward approach for comparing and assessing LSNA calibrations.

## II. THE CALIBRATION-COMPARISON METHOD APPLIED TO LARGE-SIGNAL NETWORK ANALYZERS

Conventional LSNA calibrations calculate the corrected forward-wave and backward-wave amplitude coefficients  $a_{i,calj}$  and  $b_{i,calj}$  at port  $i$ , as determined by calibration  $j$ , from the measured forward-wave and backward-wave amplitude coefficients  $a_{i,raw}$  and  $b_{i,raw}$  at port  $i$  with the relations [8;9]

$$\begin{bmatrix} a_{1,calj} \\ b_{1,calj} \\ a_{2,calj} \\ b_{2,calj} \end{bmatrix} = \begin{bmatrix} X_j & \text{[Error Box]} \\ \text{[Error Box]} & Y_j \end{bmatrix} \begin{bmatrix} a_{1,raw} \\ b_{1,raw} \\ a_{2,raw} \\ b_{2,raw} \end{bmatrix}, \quad (1)$$

where  $X_j$  and  $Y_j$  are two-by-two calibration matrices determined by the  $j^{\text{th}}$  LSNA calibration. Although crosstalk is ignored in (1),  $X_j$  and  $Y_j$  correct for most other imperfections in the LSNA as well as for absolute power and phase [10].

Equation (1) can be inverted to recover the raw data from corrected measurements, and then reapplied with another calibration to re-correct those measurements. Thus it is easy to see that the wave amplitudes corrected by two calibrations are related by

$$\begin{bmatrix} a_{1,cal1} \\ b_{1,cal1} \end{bmatrix} = X_1 X_2^{-1} \begin{bmatrix} a_{1,cal2} \\ b_{1,cal2} \end{bmatrix}; \quad \begin{bmatrix} a_{2,cal1} \\ b_{2,cal1} \end{bmatrix} = Y_1 Y_2^{-1} \begin{bmatrix} a_{2,cal2} \\ b_{2,cal2} \end{bmatrix}. \quad (2)$$

The norm  $\|A\|$  of a matrix  $A$  satisfies  $\|A\| = \max(\|Ax\|/\|x\|)$ , where  $x$  is any vector in the space spanned by  $A$  with  $\|x\| > 0$  [11]. Using this relation with (2), we derive the approximate bounds

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$$\Delta_1 \equiv \frac{\begin{bmatrix} a_{1,\text{cal1}} - a_{1,\text{cal2}} \\ b_{1,\text{cal1}} - b_{1,\text{cal2}} \end{bmatrix}}{\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}} \lesssim \|X_1 X_2^{-1} - I\| \cong \|X_2 X_1^{-1} - I\| \quad (3)$$

and

$$\Delta_2 \equiv \frac{\begin{bmatrix} a_{2,\text{cal1}} - a_{2,\text{cal2}} \\ b_{2,\text{cal1}} - b_{2,\text{cal2}} \end{bmatrix}}{\begin{bmatrix} a_2 \\ b_2 \end{bmatrix}} \lesssim \|Y_1 Y_2^{-1} - I\| \cong \|Y_2 Y_1^{-1} - I\| \quad (4)$$

on differences between measurements calculated with the two calibrations, where  $I$  is the identity matrix, the  $a_i$  and  $b_i$  are approximate forward- and backward-wave amplitudes, and the symbol  $\lesssim$  indicate less than or approximately equal. To obtain these approximate expressions, we assumed that the calibrations were approximately equal so that  $a_i \approx a_{i,\text{cal1}} \approx a_{i,\text{cal2}}$ ,  $b_i \approx b_{i,\text{cal1}} \approx b_{i,\text{cal2}}$ ,  $\|X_1 X_2^{-1} - I\| \approx \|X_2 X_1^{-1} - I\|$  and  $\|Y_1 Y_2^{-1} - I\| \approx \|Y_2 Y_1^{-1} - I\|$ . Thus (3) and (4) can be used when differences between calibrations are small (*e.g.* due to drift), but will fail when differences are large.

### III. TEST-SET DRIFT

Test-set drift is one of the most common errors measured with the calibration comparison method. Figures 1-2 show how the metrics  $\|X_1 X_2^{-1} - I\|$  and  $\|Y_1 Y_2^{-1} - I\|$ , which capture differences in the power, phase, and scattering-parameter calibrations of the LSNA, evolved over a 48 hour period in our instrument. Here, calibration 1 was the first calibration we performed, while calibration 2 was performed some time later. In this experiment, the differences between the calibrations first grew larger as the test set drifted from its initial state with time, but later returned to the state shown in Fig. 2, which was somewhat nearer to the initial state than the worst-case deviations we measured in this 48-hour period.

Figures 1 and 2 also compare the differences  $\Delta_1$  and  $\Delta_2$  for an amplifier operating in saturation to the metrics  $\|X_1 X_2^{-1} - I\|$  and  $\|Y_1 Y_2^{-1} - I\|$  for the calibrations. As expected, the metrics  $\|X_1 X_2^{-1} - I\|$  and  $\|Y_1 Y_2^{-1} - I\|$  bound the differences  $\Delta_1$  and  $\Delta_2$ .

More importantly, the metrics  $\|X_1 X_2^{-1} - I\|$  and  $\|Y_1 Y_2^{-1} - I\|$  are not very much larger than  $\Delta_1$  and  $\Delta_2$ . This shows that the upper bounds we derive on measurement differences are not overly conservative upper bounds and can be very nearly attained in practice. This is a consequence of using the matrix norm, which is always attained by some vector in the space. This makes these bounds not just useful as indicators of worst-case measurement deviations, but also useful as gauges of actual deviations that might be expected in practice.

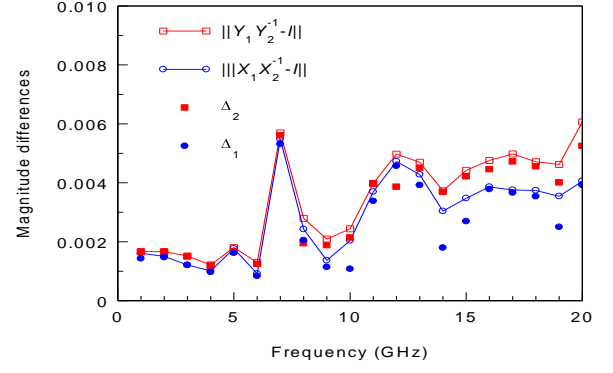


Fig. 1. Effect of test-set drift on calibrations and amplifier measurements after one-half hour.

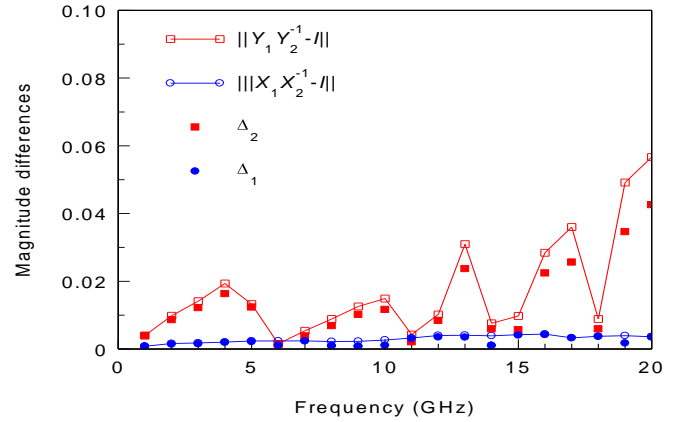


Fig. 2. Effect of test-set drift on calibrations and amplifier measurements after two days.

### IV. POWER, PHASE, AND SCATTERING-PARAMETERS

LSNA calibrations can be viewed as a combination of three calibrations, a power calibration based on microwave power incident on a calibrated power meter, a phase calibration based on measurements of a calibrated comb generator, and a scattering-parameter calibration based on a scattering-parameter calibration kit. The matrices  $X_j$  and  $Y_j$  contain separate information on how these three aspects of the two LSNA calibrations are related.

These portions of the calibrations can be examined separately by rewriting the matrices  $X_j$  and  $Y_j$  as

$$\begin{bmatrix} X_j & \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} \\ \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} & Y_j \end{bmatrix} = K_j \begin{bmatrix} X'_j & \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} \\ \begin{bmatrix} \bullet \\ \bullet \end{bmatrix} & Y'_j \end{bmatrix}, \quad (6)$$

where the  $K_j$  are chosen so that  $X_{1,11}' = X_{2,11}' = 1$ . The magnitudes of the  $K_j$  are determined from a measurement of the calibrated power meter, and provide information on differences in the two power calibrations. The phases of the  $K_j$  are determined from the comb generator measurements, and provide information on the progression of the phases of the forward and backward waves as a function of frequency.

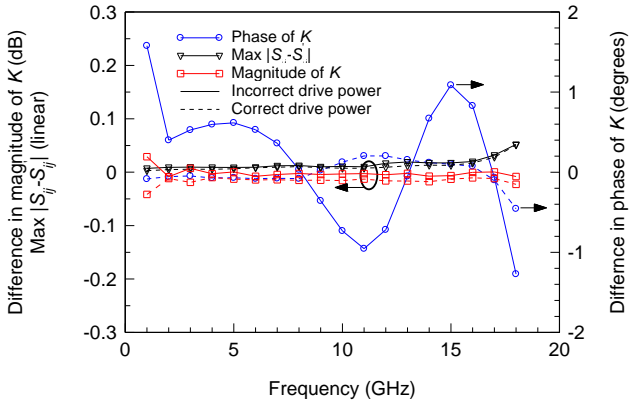


Fig. 3. Comparison of calibrations with no difference in comb-generator drive power (dashed) and with large differences in comb-generator drive power (solid). We removed a linear term from the differences of the phases of  $K$  that minimizes differences in a least-squares sense. This is equivalent to shifting the time reference of the two calibrations.

Finally, the  $X'_j$  and  $Y'_j$  are determined by the scattering-parameter portion of the calibration. These matrices allow the metric  $\max |S_{ij}-S'_{ij}|$ , which bounds differences of the scattering-parameters  $S_{ij}$  and  $S'_{ij}$  of linear passive devices measured by two calibrations, to be determined from the  $X'_j$  and  $Y'_j$  by use of the formulae in [1].

To illustrate the usefulness of separately examining these different metrics, we calibrated our LSNA in the standard way and compared it to a second calibration in which we deliberately reduced the drive power to the comb generator by 0.4 dB, an amount much greater than the typical  $\pm 0.25$  dB specified by the manufacturer [12]. While small changes in the drive power supplied to the comb generator during calibration have little effect on the calibrations [12], we deliberately introduced this significantly large change of drive power to obtain easily measured differences between the calibrations.

The solid curves in Fig. 3 show how this large difference in the drive power supplied to the comb generator changed the calibration. The curve marked with squares shows the effect of this change in drive power on the magnitude of  $K$ , which changes by less than 0.02 dB. The curve marked with circles shows the change in the phase of  $K$ , which is on the order of  $\pm 1$  degree; it is possible that the jump in the phase at the lowest frequency point is due to a change in the direct coupling of the 1 GHz comb-generator drive signal to the output of the comb generator. Finally, the curve marked with triangles shows the conventional metric  $\max |S_{ij}-S'_{ij}|$  determined from the  $X'_j$  and  $Y'_j$  by use of the formulae in [1]. This curve shows that the difference of the scattering parameters of passive devices measured with the two calibrations differ by less than 0.05 [4].

To better illustrate which of these differences are significant, we performed a third calibration in which the comb-generator drive power was returned to its initial setting. The dashed curves in the figure show the differences between this calibration and the initial calibration, and are due to test-set drift, as opposed to changes in comb-generator drive

power.

Figure 3 shows that a large change in the comb-generator drive power has a larger effect on the phase of  $K$  than can be explained by test-set drift in the experiment. The figure also shows that the change in the magnitude of  $K$  and the conventional metric  $\max |S_{ij}-S'_{ij}|$  of [4] are similar in both cases, thereby confirming our expectation that even large changes in the drive power to the comb generator, such as those used in our illustration, do not introduce significant systematic error in the power or scattering-parameter portions of the LSNA calibration.

## V. SOFTWARE

We have developed an easy-to-use freeware package [13] to simplify calculations of the norms  $\|X_1 X_2^{-1} - I\|$  and  $\|Y_1 Y_2^{-1} - I\|$ , the changes in the magnitude and phase of  $K$ , and the conventional metric  $\max |S_{ij}-S'_{ij}|$  of [4]. The software also allows specific measurements to be corrected with two different calibrations and compared.

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